SERIES CONVERGENCE & DIVERGENCE FLOWCHART

(Follow the steps to determine if a series converges or diverges!)

Step 1: Check the Basics

Is the series of the form $\sum a_n$? If yes, continue!

Does a_n approach zero as $n \to \infty$?

If **NO** → The series diverges by the Divergence Test.

If **YES** → Continue to Step 2.

Step 2: Identify the Type of Series

Is it a p-series?
(Where p is a constant.) $\sum \frac{1}{n^p}$

If $p > 1$ → Converges.

If $p \leq 1$ → Diverges.

Is it a geometric series?
(Where r is the common ratio) $\sum ar^n$

If $|r| < 1$ → Converges.

If $|r| \geq 1$ → Diverges.

Is it an alternating series?

If $\left\{ \begin{array}{l} a_n \text{ decreases} \\ \lim_{n \to \infty} a_n = 0 \end{array} \right\}$ Converges.

Otherwise, try another test!

Step 3: Use the Right Convergence Test

Comparison Test
Try this if your series looks similar to a known p-series or geometric series.

Limit Comparison Test
Use this if direct comparison doesn't work.

Ratio Test
Best for factorials, exponentials, or powers.

Root Test
Good for nth powers..

Integral Test
Use for series that resemble improper integrals.

Step 4: Absolute vs Conditional Convergence

(If Alternating Series Passed)

If $\sum a_n$ **converges → Absolutely Convergent** (which means the original series also converges).

If $\sum a_n$ **diverges, but** $\sum a_n$ **converges → Conditionally Convergent.**

Summary: Which Test to Use?

P-Series / Geometric Series? → Check direct rules.
Alternating Series? → Try the Alternating Series Test first.
Factorial, exponential, or power expressions? → Try the Ratio or Root Test.
Integral-looking terms? → Try the Integral Test.
Need comparison? → Try the Comparison or Limit Comparison Test.
Not sure? → Use Ratio or Root Test as general go-tos!

THE RIGHT CONVERGENCE TEST
(For step 3)

Comparison Test

MATHIGH

Compare a_n with a known convergent/divergent series.

If $0 \le a_n \le b_n$ and $\sum b_n$ converges then, $\sum a_n$ converges.

If $b_n \ge a_n$ and $\sum b_n$ diverges then, $\sum a_n$ diverges.

Limit Comparison Test

Pick a known series $\sum b_n$ and compute $\lim_{n\to\infty} \frac{a_n}{b_n} = L$.

If $0 < L < \infty$, both series behave the same (both converge or both diverge).

Ratio Test

Compute $L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

If $L < 1$, the series converges absolutely.

If $L > 1$ or $L = \infty$, the series diverges.

If $L = 1$, the test is inconclusive—try another method!

Root Test

Compute $L = \lim_{n\to\infty} \sqrt[n]{|a_n|}$

If $L < 1$, the series converges absolutely.

If $L > 1$, the series diverges.

If $L = 1$, the test is inconclusive.

Integral Test

MATHIGH

If $f(x)$ (continuous, positive, decreasing) and $\int_1^\infty f(x)\,dx$ converges, then $\sum a_n$ also converges.

If $\int_1^\infty f(x)\,dx$ diverges then $\sum a_n$ also diverges.