MATHIGH

AP Calculus Quick Concept Notes

🧭 Overview

Your complete conceptual guide to mastering **AP Calculus AB & BC** — built for understanding, not memorization.
Clear explanations, key formulas, visual logic, and real exam-style reasoning.

🔹 Differentiation — Understanding Instantaneous Change

Core Idea: The derivative represents *instantaneous rate of change* — the slope of the tangent line.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Essential Rules:

- **Power:** $\frac{d}{dx}[x^n] = nx^{n-1}$
- **Product:** $(uv)' = u'v + uv'$
- **Quotient:** $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$
- **Chain:** $(f(g(x)))' = f'(g(x)) \cdot g'(x)$
- **Implicit / Inverse Differentiation**
- **Parametric:** $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

Key Theorem — Mean Value Theorem (MVT):
If f is continuous on [a,b] and differentiable on (a,b), then there exists some c ∈ (a,b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

MVT guarantees that the *instantaneous* rate of change equals the *average* rate somewhere in the interval.

Applications: Tangent/normal lines, motion, optimization, related rates.

Common Pitfalls:
⛔ Forgetting the inner derivative in Chain Rule.
⛔ Confusing local vs absolute extrema.

🔹 Integration & Accumulation — The Total Change Principle

Core Idea: Integration is the inverse of differentiation; it measures *total change or accumulated area*.

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \ \ (n \neq -1)$$

Fundamental Theorem of Calculus (FTC):

- **Part I (Net Change):** $\int_a^b f'(x)\,dx = f(b) - f(a)$
- **Part II (Derivative of Integral):** $\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$

Applications:

- Area Between Curves
- Volume (Disk / Washer / Shell)
- Accumulation in motion models: $s(t) = \int v(t)\,dt$

BC Extensions:

- **Integral Test** (for series):
  If f(x) is positive, decreasing, and continuous, then
  $\sum f(n)$ converges iff $\int f(x)\,dx$ converges.
- **Polar Area:**
  $A = \frac{1}{2}\int_\alpha^\beta [r(\theta)]^2 \, d\theta$

Common Mistakes:

⛔ Missing constant of integration.
⛔ Using wrong bounds or variable.

◆ Differential Equations — Modeling Continuous Change

Core Idea: Express how rates describe dynamic systems.

Forms:

- **Separable:** $\frac{dy}{dx} = ky \Rightarrow y = Ce^{kx}$
- **Logistic Model:** $\frac{dy}{dx} = ky\left(1 - \frac{y}{L}\right)$
- **Slope Fields:** Visualize behavior of solutions.
- **Euler's Method:** Approximate numerically.

BC Extension — Series Solutions:

- **Taylor Series:** $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$
- **Maclaurin Series (c=0):** $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n$
- **Error Bound (Lagrange):** $R_n(x) \leqslant \frac{M|x-c|^{n+1}}{(n+1)!}$

Tip: Always interpret DEs graphically before solving.

◆ Applications of Derivatives — Analyzing Behavior

Core Idea: Derivatives describe how functions grow, curve, and reach extremum values.

Tests & Theorems:

- **Critical Points:** where f'(x)=0 or undefined
- **First Derivative Test:** f' changes + → – ⇒ max; – → + ⇒ min
- **Second Derivative Test:** f"(c)>0 ⇒ min; f"(c)<0 ⇒ max
- **Concavity:** f">0 ⇒ concave up; f"<0 ⇒ concave down
- **Inflection Points:** where f" changes sign
- **Extreme Value Theorem:** continuous f on [a,b] has both min & max

Optimization Framework:

1. Define the quantity to optimize.
2. Express it in one variable.
3. Differentiate → set f'(x)=0.
4. Verify with sign or endpoint test.

MVT in Practice: Connects slope of secant to tangent — a foundational FRQ justification.

Common Mistakes:
⛔ Ignoring endpoints.
⛔ Forgetting justification (“since f' changes sign…”).

🔹 Final Thoughts — Mathematical Communication

AP Calculus is not just computation — it’s reasoning.
Scoring is built on *conceptual clarity and justification*.

“The exam doesn’t ask if you know the formula —
it asks if you can explain why it works.”

Follow this sequence:
Claim → Reason → Evidence

Be precise, be visual, be logical.