🧮 AP Calculus BC Formula Sheet (Final Elite Edition)

Complete reference for the AP Calculus BC Exam – includes all AB content + BC-exclusive topics. Designed for the digital and paper exam formats.

I. Core AB Review (Foundation for BC)

- **Limits & Continuity:** All standard limit laws, indeterminate forms, and continuity criteria apply.
- **Derivative Techniques:** Power, Product, Quotient, Chain, Implicit, Inverse, Exponential, Logarithmic, and Trig Derivatives.
- **Integrals & FTC:**

$$\int_a^b f'(x)\,dx = f(b) - f(a),\ \frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$$

- **Motion Models:**

$$v(t) = s'(t),\ a(t) = v'(t),\ s(t) = \int v(t)\,dt$$

II. Integration Techniques (BC-Exclusive Extensions)

🔹 **Integration by Parts**

$$\int u\,dv = u\,v - \int v\,du$$

Choose $u$and dvusing **LIATE rule** (Logarithmic → Inverse → Algebraic → Trig → Exponential).

🔹 **Partial Fractions**

Decompose rational functions:

$$\int \frac{P(x)}{Q(x)}dx = \int \left(\frac{A}{x-a} + \frac{B}{x-b}\right)dx \Rightarrow A\ln|x-a| + B\ln|x-b|$$

Example:

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$$

🔹 **Improper Integrals**

Check convergence using limits:

$$\int_1^\infty \frac{1}{x^p}dx \text{ converges if } p > 1$$

III. Sequences & Series

◆ Convergence Tests Summary

Test	Condition	Converges When
n-th Term (Divergence)	$\lim_{n\to\infty} a_n \neq 0$	Diverges
Geometric	$\sum a \cdot r^n$	$\lvert r\rvert < 1$
p-Series	$\sum \frac{1}{n^p}$	$p > 1$
Alternating Series Test	$a_n > 0,\ a_{n+1} \le a_n,\ \lim a_n = 0$	Conditionally
Ratio Test	$L = \lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert$	$\begin{cases} \text{Converges if } L<1 \\ \text{Diverges if } L>1 \\ \text{Inconclusive if } L=1 \end{cases}$
Root Test	$L = \lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert}$	$\begin{cases} \text{Converges if } L<1 \\ \text{Diverges if } L>1 \\ \text{Inconclusive if } L=1 \end{cases}$
Integral Test	*f(x) positive, continuous, decreasing; $a_n = f(n)$*	$\int_1^\infty f(x)\,dx$ converges
Comparison Test	$0 \le a_n \le b_n$	$\sum b_n$ converges ⇒ $\sum a_n$ converges
	$0 \le b_n \le a_n$	$\sum b_n$ **diverges** ⇒ $\sum a_n$ **diverges**
Limit Comparison Test **(LCT)**	$\lim_{n\to\infty} \frac{a_n}{b_n} = L,\ 0 < L < \infty$	Both $\sum a_n$ and $\sum b_n$ behave the same

Error Bound (Alternating Series): $\lvert R_n \rvert \le \lvert a_{n+1} \rvert$

IV. Power, Taylor & Maclaurin Series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!} (x-c)^n$$

$$R_n(x) = \frac{M|x-c|^{n+1}}{(n+1)!}$$

Radius of Convergence: $|x-c| < R$ from Ratio Test.

Common Maclaurin Series (for |x|<1 unless stated)

Function	Expansion
$\frac{1}{1-x}$	$\sum_{n=0}^{\infty} x^n$
e^x	$\sum_{n=0}^{\infty} \frac{x^n}{n!}$
$\sin x$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$
$\cos x$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$
$\arctan x$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$
$\ln(1+x)$	$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$
$(1+x)^k$	$\sum_{n=0}^{\infty} \binom{k}{n} x^n$ (Generalized Binomial)

Series Manipulation Tips

- **Multiply by x^m to shift powers (e.g. $x\cos x$).**
- **Substitute $x \to 2x$ for expansions like e^{2x}.**
- **Differentiate or integrate known series to generate new ones.**

V. Parametric & Polar Functions

- **Parametric**

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}, \quad \frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy/dx}{dx/dt}\right)$$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

- **Polar**

$$x = r\cos\theta,\ y = r\sin\theta$$

$$\frac{dy}{dx} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$$

$$A = \frac{1}{2}\int_\alpha^\beta [r(\theta)]^2 d\theta,\ L = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

VI. Vector Functions

Concept	***Formula***
Position	$\vec{r}(t) = \langle x(t), y(t) \rangle$
Velocity	$\vec{v}(t) = \vec{r}'(t)$
Acceleration	$\vec{a}(t) = \vec{v}'(t)$
Speed	$\lvert\vec{v}'(t)\rvert$
Distance	$\int_a^b \lvert\vec{v}(t)\rvert\, dt$
Reconstruct Position	$\vec{r}(t) = \vec{r}(a) + \int_a^t \vec{v}(u)\, du$

VII. Exam-Day Strategies

✅ FRQ Strategy

- Always write the **first 2–3 terms** explicitly in Taylor or Maclaurin expansions. Partial credit is awarded even if the remainder is wrong.
- Label your reasoning ("Since $f'(x)$ changes from + to –, f has a local max at x = ...").

✅ Error Prevention

- Polar Area → never forget the ½ and r^2.
- In vector/parametric problems, clearly separate derivatives wrt t and x.

✅ Time Management

- If a calculator question takes >4 minutes, approximate numerically (≈) and move on.
- Show each substitution step in partial fractions / integration by parts for rubric credit.