

2026

6

AP[®] Calculus BC

Full-Length Practice Exams

- 6 Full-Length Exams
- Answer Keys Included
- QR: Solutions + FRQ Scoring Guides

FREE PREVIEW EDITION

PRECISION • MASTERY • CONFIDENCE



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MATHIGH

AP CALCULUS BC — PRACTICE EXAM

Full-Length Exams • MCQ + FRQ • Real AP Style

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QUICK START

Welcome! This book contains **6 full-length AP Calculus BC practice exams** written in real AP style.

1) Choose the right exam today

- **Practice Exam 1 — Baseline:** First checkpoint. Find your starting level.
- **Practice Exam 2 — Core Skills:** Strengthen the essentials and consistency.
- **Practice Exam 3 — Balanced AP Exam:** Closest to the overall AP balance.
- **Practice Exam 4 — Threshold:** Separate strong 4-level from potential 5-level.
- **Practice Exam 5 — Challenge:** Higher difficulty for top-score preparation.
- **Practice Exam 6 — Final Mock:** Your final rehearsal (take 7–10 days before exam day).

2) Take the exam the right way

- Follow the **official timing** for each section.
- Use a **quiet setting** and take only the standard break between sections.
- **Do not** use notes, formula sheets, or solution videos while testing.
- If stuck, **make your best attempt and move on**—time management is part of the score.

3) Score it like a pro

- Each exam includes:
 - **MCQ Answer Key**
 - **FRQ Final Answers**
 - **FRQ Scoring Guidelines**
 - **Score Interpretation (1–5 estimate)**
- Score MCQ and FRQ separately first, then combine for your overall estimate.

4) The 10-minute review rule (most important)

After scoring, spend **10 minutes** doing only this:

1. Write the question numbers you missed.
2. Label the reason: **Concept / Setup / Algebra / Calculator / Justification / Time**
3. Pick **2 topics** to fix before the next exam.

One goal: If you take the exams in order and review mistakes carefully, you'll build the habits that typically lead to higher scores.

HOW TO USE THIS BOOK

This book is built around one idea: **your score improves fastest when you practice under real conditions and review mistakes correctly.**

You will take **6 full-length exams**, each with a specific purpose.

1) Take the exams in order

Each practice exam has a unique role:

- **Exam 1 — Baseline:** Find your starting point and pacing habits.
- **Exam 2 — Core Skills:** Build consistency on essential BC skills.
- **Exam 3 — Balanced:** A realistic indicator of where you stand now.
- **Exam 4 — Threshold:** Identify what is blocking a 5-level score.
- **Exam 5 — Challenge:** Train stamina and high-level decision making.
- **Exam 6 — Final Mock:** Your final rehearsal (take 7–10 days before exam day).

2) Use real AP testing conditions

For the most accurate results:

- Follow the official **Section I / Section II timing**.
- Use only permitted tools (calculator only when allowed).
- Do not pause the test, check answers, or redo problems mid-exam.
- If stuck, make your best attempt and move on—**time management is part of the test**.

3) Score it the right way

After finishing an exam:

1. Check your **MCQ answers**.
2. Score each **FRQ out of 9 points** using the scoring guidelines.
3. Combine your MCQ + FRQ results to get an estimated **AP score (1–5)**.
4. Compare MCQ and FRQ performance separately to see whether your main issue is:
 - content gaps,
 - execution (algebra/notation), or
 - strategy (time, reading, justification).

4) The most important step: review mistakes

Your score does not improve during the exam—it improves **after** the exam.

Use the **Error Log** and classify each mistake as one of these:

- Concept / Setup / Algebra / Calculator / Justification / Time

Then choose two topics to fix before the next exam.

Do focused practice, then move on to the next full-length test.

AP CALCULUS BC EXAM FORMAT

The AP Calculus BC Exam is a full-year college-level calculus exam, covering all AB topics **plus additional BC-only content** such as advanced integration techniques, parametric and polar analysis, sequences and series, and Taylor polynomials.

The exam is **3 hours and 15 minutes** long and consists of two main sections.

Section I — Multiple Choice (MCQ)

Total time: 1 hour 45 minutes

- **Part A: No Calculator**

30 questions — 60 minutes

Focus: algebraic fluency, limits, derivatives, integrals, graphs/tables, and reasoning without technology.

- **Part B: Calculator Allowed**

15 questions — 45 minutes

Focus: numerical evaluation, modeling, interpreting tables/graphs, and verifying results efficiently.

MCQ Scoring: Each question is worth **1 point**. No penalty for incorrect answers.

Section II — Free Response (FRQ)

Total time: 1 hour 30 minutes

- **Part A: Calculator Allowed**

2 questions — 30 minutes

Focus: using technology appropriately (numerical solutions, integrals, regression/modeling).

Clear reasoning and correct units still matter.

- **Part B: No Calculator**

4 questions — 60 minutes

Focus: symbolic work, exact values, reasoning, and written justification.

FRQ Scoring: Each FRQ is scored on a **0–9 point rubric**.

Partial credit is awarded for correct method and communication, even if arithmetic is imperfect.

BC-Specific Notes

- The BC exam includes all AB topics, but places greater emphasis on:

- Parametric, polar, and vector-valued functions
- Sequences and series (including convergence tests)
- Taylor and Maclaurin polynomials

- Students who perform well may earn:

A score of 5 on AP Calculus BC, and an AB subscore, which many universities recognize separately.

SCORE GUIDE (UNOFFICIAL ESTIMATE)

Important: College Board does not publish official “raw-to-AP score” cutoffs, and thresholds can shift from year to year.

This guide gives a **reasonable estimate** using a widely used **50/50 MCQ–FRQ** model. Use it to track progress—not as a guarantee.

How scoring works (quick)

- **MCQ:** 45 questions → **MCQ Correct = 0–45** (no penalty for wrong)
- **FRQ:** 6 questions, each **0–9** → **FRQ Total = 0–54**

How to compute your Composite (0–108)

1. MCQ Scaled (0–54)

$$\text{MCQ Scaled} = \text{round}(1.2 \times \text{MCQ Correct})$$

2. FRQ Total (0–54)

$$\text{FRQ Total} = \text{FRQ1} + \text{FRQ2} + \text{FRQ3} + \text{FRQ4} + \text{FRQ5} + \text{FRQ6}$$

3. Composite (0–108)

$$\text{Composite} = \text{MCQ Scaled} + \text{FRQ Total}$$

Scan to calculate instantly

Use the QR code to enter **MCQ correct + FRQ1–FRQ6** and get:

MCQ Scaled, FRQ Total, Composite, and a score estimate.

“Calculate your score in 10 seconds (MCQ + FRQ).”



Backup URL: <https://mathhigh.com/ap-calculus-ab-score-calculator/>

PRACTICE EXAM REPORT SHEET

Student Name: _____	Exam: <input type="checkbox"/> 1 <input type="checkbox"/> 2 <input type="checkbox"/> 3 <input type="checkbox"/> 4 <input type="checkbox"/> 5 <input type="checkbox"/> 6
Date Taken: / /	Start Time: End Time:
Testing Conditions: <input type="checkbox"/> Full (Official Timing) <input type="checkbox"/> Paused <input type="checkbox"/> Untimed Other: _____	
Calculator Use (when allowed): <input type="checkbox"/> Appropriate <input type="checkbox"/> Overused <input type="checkbox"/> Underused <input type="checkbox"/> Not sure	

A) SCORE SUMMARY (Fill in after grading)

MCQ Correct: ____ / 45

MCQ Scaled: ____ / 54 (round($1.2 \times$ MCQ Correct))

FRQ Scores (each /9):

FRQ1 ____ FRQ2 ____ FRQ3 ____ FRQ4 ____ FRQ5 ____ FRQ6 ____

FRQ Total: ____ / 54

Composite: ____ / 108 (MCQ Scaled + FRQ Total)

Estimated AP Score (from QR calculator): 1 2 3 4 5

B) SECTION DIAGNOSIS (Circle ONE)

- MCQ is limiting my score
- FRQ is limiting my score
- Time management is limiting my score
- Balanced — I need overall improvement

C) POINT-LOSS MAP (Circle your TOP 2 causes)

Concept • Setup • Algebra • Justification • Units/Rounding • Time/Strategy • Calculator Misuse

D) QUICK NOTES (Student Reflection)

What felt hardest?

What improved compared to the last exam?

One mistake I will never repeat:

AP Calculus BC - Practice Exam 1

~ Baseline Exam ~

How to Use This Exam

This exam is designed as a **starting point**.

Do **not** expect it to feel overwhelming.

How to take it

Take the exam under timed conditions, but without pressure.

If you are unsure about a question, make your best attempt and move on.

Do not review notes or formulas during the exam.

What this exam measures

Whether you have a solid foundation of AP Calculus AB concepts

Which topics are secure and which ones need review

Your initial pacing and time management

How to use your score

This exam is not a predictor of your final AP score.

Use it to create a targeted review plan.

Focus on patterns of mistakes rather than the raw score.

This exam answers one question:

“Am I ready to begin serious AP exam preparation?”

“The journey of a thousand miles begins with a single step.”

Lao Tzu

Section I: Multiple-Choice Questions

Directions: Solve each of the following questions carefully. You may use the space provided in the test booklet for scratch work as you work through each problem.

After reviewing the answer choices, select the one that best answers the question. Try to manage your time effectively and avoid spending too much time on any single problem.

In this test: Unless otherwise stated, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Section I, Part A

Time - 60 minutes, Number of questions - 30

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS

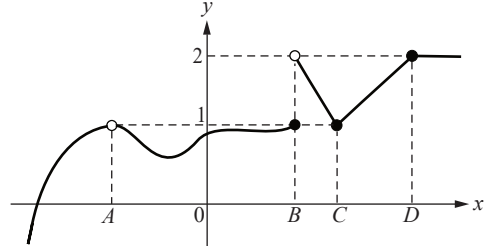
1. If $f'(x) = 4x^2 - 6x$ and $f(3) = 5$, what is the value of $f(0)$?

- A) -4
- B) -2
- C) 0
- D) 5

2. If $y = x^2(e^x - 2)$, then $\frac{dy}{dx} =$

- A) $2xe^x$
- B) $2x(e^x - 2)$
- C) $x^2e^x + 2xe^x - 4x$
- D) $x^2e^x + 2xe^x - 2x^2 - 4x$

3. The graph of a function f is shown. The x -values labeled A , B , C and D are marked on the graph.



At which labeled x -values does $\lim_{x \rightarrow a} f(x)$ exist but f is not continuous at $x = a$?

- A) A
- B) B
- C) C
- D) D

MATH101

4. A particle moves in the plane so that its position at time t is given by

$$x(t) = t^2 - 1 \quad \text{and} \quad y(t) = 3t + 2.$$

What is the value of $\frac{dy}{dx}$ when $t = 1$?

- A) $\frac{1}{3}$
- B) $\frac{3}{2}$
- C) 2
- D) 3

5. Evaluate $\int \frac{1}{x^2 + x - 2} dx$

A) $\ln|x - 1| + \ln|x + 2| + C$

B) $\frac{1}{3}\ln|x - 1| - \frac{1}{3}\ln|x + 2| + C$

C) $\ln\left|\frac{x-1}{x+2}\right| + C$

D) $\frac{1}{x-1} - \frac{1}{x+2} + C$

6. Which of the following series converges?

A) $\sum_{n=1}^{\infty} \frac{1}{n}$

B) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

C) $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$

D) $\sum_{n=1}^{\infty} \frac{n}{n+1}$

7. A particle moves along the x -axis so that for all $t \geq 0$, its acceleration is $a(t) = 4\sin(2t)$. If the velocity of the particle at $t = 0$ is $v(0) = 3$ and its position at $t = 0$ is $x(0) = 1$, which of the following gives the position of the particle at time t ?

A) $-\sin(2t) + 5t + 1$

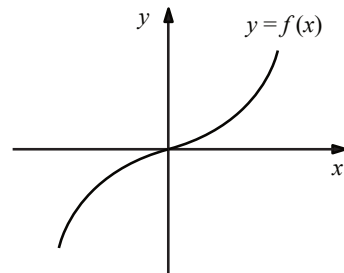
B) $\sin(2t) + 5t + 1$

C) $-2\cos(2t) + 5t + 1$

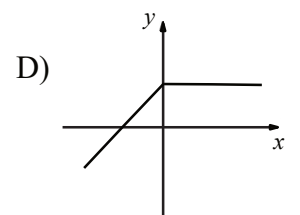
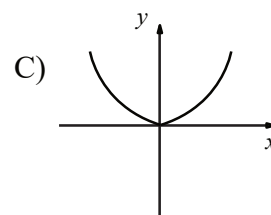
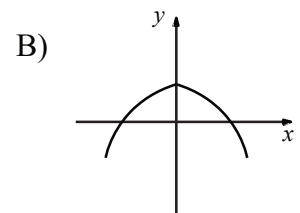
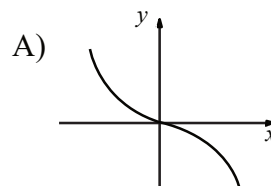
D) $2\cos(2t) + 5t + 1$

MATHIGH

8.



The graph of the function $f(x)$ is given in the figure. Based on this, which of the following could be the graph of the derivative $f'(x)$?



9. Let f be a function such that

$$f(2) = 5, f'(2) = 3, f''(2) = 4.$$

Using the second-degree Taylor polynomial for f about $x = 2$, the approximation of $f(2.1)$ is

- A) 5.3
- B) 5.31
- C) 5.32
- D) 5.33

10. If $\frac{dy}{dx} = x^4 - 3x^3 + 2x - 5$, then $\frac{d^2y}{dx^2}$ evaluated at $x = 1$ is equal to:

- A) -3
- B) 2
- C) 6
- D) 12

11. Let f be the function defined by

$$f(x) = \begin{cases} x^2, & x < 0 \\ -2, & x = 0 \\ x, & x > 0 \end{cases}$$

What is $\int_{-1}^1 f(x) dx$?

- A) $-\frac{5}{6}$
- B) $-\frac{1}{6}$
- C) $\frac{1}{6}$
- D) $\frac{5}{6}$

MATHCH

12. A curve is given in polar form by $r = 2\cos\theta$. What is the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{8}$?

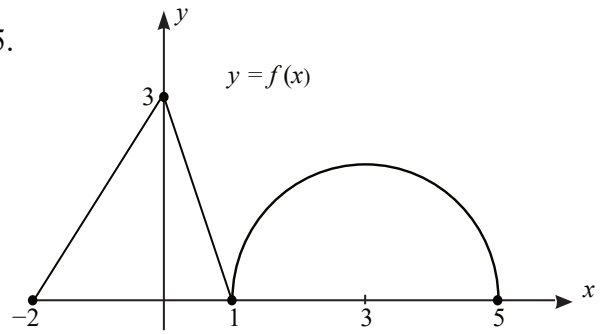
- A) -1
- B) $-\frac{1}{2}$
- C) $\frac{1}{2}$
- D) 1

13. For time $t \geq 1$, the position of a particle moving along the x -axis is given by $p(t) = \sqrt{t} + 1$. At what time t in the interval $1 \leq t \leq 9$ is the instantaneous velocity of the particle equal to the average velocity of the particle over the interval $1 \leq t \leq 9$?

- A) 1
B) 4
C) 6
D) 9

14. Let f be a differentiable function such that $f(4) = 1$ and $f'(4) = 5$. If $y = \sin(f(x^2))$, what is the value of $\frac{dy}{dx}$ when $x = 2$?

- A) $5\cos(1)$
B) $4\cos(1)$
C) $20\cos(1)$
D) $10\cos(1)$



The graph of the function f shown consists of two line segments and a semicircle. Let g be defined by $g(x) = \int_0^x f(t) dt$. What is the value of $g(5) = ?$

- A) 0
B) $-1.5 + 2\pi$
C) $1.5 + 2\pi$
D) $4.5 + 2\pi$

MATHCH

16. Let f be the function given by

$$f(x) = x^3 - 6x^2 + 9x.$$

What is the maximum value of f on the interval $[0, 4]$?

- A) 0
B) 4
C) 6
D) 8

17. For which values of p does the integral

$$\int_1^{\infty} \left(\frac{1}{x^p}\right) dx$$

converge?

- A) $p > 0$
 - B) $p \geq 0$
 - C) $p > 1$
 - D) All real values of p
18. Let f be the function defined by $f(x) = x\sqrt{x}$. What is the approximation for $f(5)$ found by using the line tangent to the graph of f at the point $(4, 8)$?
- A) 3
 - B) 5
 - C) 10
 - D) 11

19. Evaluate $\lim_{x \rightarrow 0} \frac{3x^2}{e^{3x} - 3x - 1}$

- A) 0
- B) $\frac{1}{2}$
- C) $\frac{2}{3}$
- D) 1

MATHCH

20. Let f be a twice-differentiable, increasing function on the interval $0 \leq x \leq 8$. If $f(0) = 5$ and $f(8) = 69$, which of the following must be true on the interval $0 < x < 8$?

- A) $f'(x) = 0$ for some x in the interval
- B) $f'(x) = 8$ for some x in the interval
- C) $f''(x) = 0$ for some x in the interval
- D) $f''(x) > 0$ for all x in the interval

21. Evaluate $\frac{d}{dx} \left(\int_1^{x^4} \ln(t^2 + 3) \cdot dt \right)$.

- A) $\ln(x^8 + 3)$
- B) $4x^3 \ln(x^8 + 3)$
- C) $4x^3 \ln(x^4 + 3)$
- D) $\ln(x^8 + 3) - \ln(4)$

22. Using the substitution $u = x + 2$, the integral

$$\int \frac{x}{\sqrt{x+2}} dx$$

is equivalent to which of the following?

- A) $\int \frac{1}{u+2} du$
- B) $\int \frac{1}{\sqrt{u}} du$
- C) $\int \left(\sqrt{u} - \frac{2}{\sqrt{u}} \right) du$
- D) $\int \left(\sqrt{u} - \frac{1}{\sqrt{u}} \right) du$

23. Let f be the function given by

$$f(x) = \frac{x^2 + x - 6}{x^2 - 5x - 24}$$

For which value(s) of x does f have a removable discontinuity?

- A) -3 only
- B) -2 only
- C) 3 only
- D) -2 and 3

MATHCH

24. Which of the following is a solution to the differential equation

$$\frac{dy}{dx} = 3y?$$

- A) $y = e^{3x}$
- B) $y = 3e^x$
- C) $y = \sin(3x)$
- D) $y = \cos(3x)$

25. The table gives values of a continuous function f at selected values of x .

x	2	3	4	5	6
$f(x)$	7	4	5	8	3

If f has exactly two critical points on the open interval $(2, 6)$, which of the following must be true?

- A) $f(x) > 0$ for all $x \in (2, 6)$
- B) $f'(x)$ exists for all $x \in (2, 6)$
- C) $f'(x) < 0$ for all $x \in (2, 3)$
- D) $f'(4) \neq 0$

26. The positive variables x and y change with respect to time t . They are related by the equation $x^2 = (18 - y)^3$. At the instant when $\frac{dx}{dt} = 6$ and $y = 14$, what is the value of $\frac{dy}{dt}$?

- A) -3
- B) -2
- C) 2
- D) 3

27. Let

$$\mathbf{r}(t) = \langle 3t^2, 4t \rangle$$

be the position vector of a particle. What is the speed of the particle at $t = 1$?

- A) 5
- B) $\sqrt{26}$
- C) 7
- D) $\sqrt{52}$

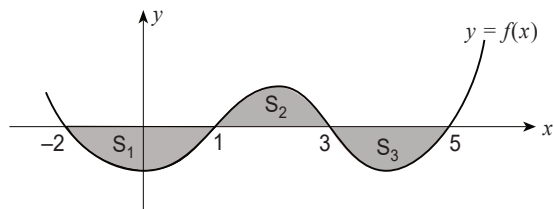
MATHCH

28. The Maclaurin series for e^x is used to approximate $e^{0.2}$.

What is the least degree n such that the error is less than 10^{-4} ?

- A) $n = 2$
- B) $n = 3$
- C) $n = 4$
- D) $n = 5$

29.



The graph of a continuous function f is shown above on the interval $[-2, 5]$. The regions bounded by the graph of f and the x -axis are labeled S_1 , S_2 , and S_3 , as shown. It is given that

$$\int_{-2}^5 f(x) dx = -6 \quad \text{and} \quad \int_{-2}^5 |f(x)| dx = 12.$$

Find the area of region S_2 .

- A) 3
- B) 6
- C) 9
- D) 12

30. Water is poured into a container with a circular base. The volume V of the water in the container is given by $V = \frac{1}{2}r^3$, where r is the radius of the base, in feet. The circumference of the base is increasing at a constant rate of 6π feet per hour. When the circumference of the base is 12π feet, what is the rate of change of the volume of the water, in cubic feet per hour?

- A) 36
- B) 72
- C) 144
- D) 162

MATH1GH

END OF PART A OF SECTION I

Section I, Part B

Time - 45 minutes, Number of questions - 15

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS

31. A particle moves along the x -axis. The velocity of the particle at time t is given by

$$V(t) = \frac{6}{t^2 + 2}$$

If the position of the particle is $x = 2$ when $t = 1$, what is the position of the particle when $t = 3$?

- A) 2.742
- B) 3.442
- C) 4.184
- D) 5.102

32. The polar curve is given by $r = 2 + \sin \theta$, for $0 \leq \theta \leq \pi$. What is the area of the region enclosed by the curve and the polar axis over this interval?

- A) 7.069
- B) 7.534
- C) 9.852
- D) 11.069

33. A particle moves in the plane so that its position at time t is $(x(t), y(t))$. Selected values are given in the table.

t	1.8	1.9	2.0	2.1	2.2
$x(t)$	3.24	3.61	4.00	4.41	4.84
$y(t)$	1.82	1.76	1.69	1.61	1.52

Using the table, what is the best approximation for

$$\left. \frac{dy}{dx} \right|_{t=2} ?$$

- A) -0.188
- B) -0.155
- C) -0.094
- D) -0.075

MATH101

34. During a storm, the depth of water in a drainage tank increases at a rate modeled by
- $$R(t) = 0.4 + \cos\left(\frac{\pi t^2}{18}\right),$$
- where t is measured in hours since the start of the storm and $R(t)$ is measured in centimeters per hour. How much does the depth of water in the tank increase from $t = 0$ to $t = 3$ hours?
- A) 2.740 cm
 B) 3.540 cm
 C) 4.065 cm
 D) 4.126 cm
35. The acceleration of a particle moving along the x -axis is given by $a(t) = (t - 6)\cos t$ for $0 \leq t \leq 6$. At what value of t is the particle's velocity increasing most rapidly?
- A) 1.571
 B) 2.836
 C) 4.712
 D) 6
36. Let $T_3(x)$ be the third-degree Taylor polynomial for $\ln x$ about $x = 1$. Using $T_3(x)$, what is the best approximation to $\ln(1.3)$?
- A) 0.254
 B) 0.264
 C) 0.276
 D) 0.300

37. If the average value of the function g over the closed interval $[1,5]$ is 4 and if $g(x) \geq 0$ for all x in $[1,5]$, what is the area of the region enclosed by the graph of $y = g(x)$, the lines $x = 1$ and $x = 5$, and the x -axis?

- A) 1
- B) 4
- C) 16
- D) 20

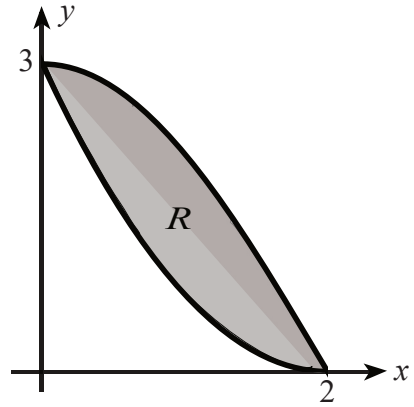
38. The function f is continuous and increasing for $x > -1$. The table below gives values of f at selected values of x .

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	-0.102	-0.01	-0.001	0.001	0.01	0.102

Which of the following is the best approximation for $2^{3f(x)}$?

- A) 0
- B) 1
- C) 2
- D) The limit does not exist.

39.



Let R be the region in the first quadrant bounded by the graphs of

$$y = 3 \cos\left(\frac{\pi x}{4}\right) \text{ and } y = \frac{3}{4}(x-2)^2$$

as shown. The region R is the base of a solid. For the solid, each cross section perpendicular to the x -axis is an isosceles right triangle with a leg in region R . What is the volume of the solid?

- A) 0.499
- B) 0.897
- C) 0.909
- D) 0.998

40. Let f be the function defined by $f(x) = \ln(x^2 + 4)$, and let g be the function defined by $g(x) = x^4 + 2x^2$. The line tangent to the graph of f at $x = 1$ is parallel to the line tangent to the graph of g at $x = a$, where a is a positive constant. What is the value of a ?

- A) 0.099
- B) 0.316
- C) 0.447
- D) 0.707

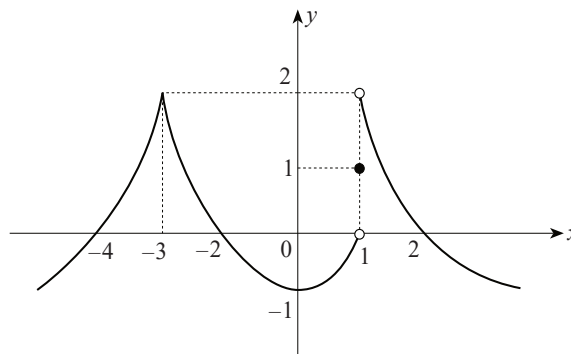
41. A function f is defined by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}.$$

Using the first 6 terms of the series, what is the best approximation for $f(0.8)$?

- A) 0.6291
- B) 0.6853
- C) 0.7126
- D) 0.7673

42.



The graph of the function $y = f(x)$ is shown above. Which of the following statements is false?

- A) $\lim_{x \rightarrow -3} f(x) = 2$
- B) $\lim_{x \rightarrow 0^-} f(x) = -1$
- C) $\lim_{x \rightarrow 1^+} f(x) = 1$
- D) $\lim_{x \rightarrow 1^-} f(x) = 0$

43. A population $P(t)$ satisfies the logistic differential equation

$$\frac{dP}{dt} = 0.6P\left(1 - \frac{P}{500}\right), P(0) = 100,$$

where t is measured in years.

Using Euler's method with step size $\Delta t = 1$, what is the best approximation for $P(3)$?

- A) 279.645
- B) 281.775
- C) 282.890
- D) 283.644

44. The temperature T , in degrees Fahrenheit ($^{\circ}F$), of a bowl of soup t minutes after it is served is given by $T(t) = 180e^{-0.08t} + 70$, $t \geq 0$.

To the nearest degree, what is the average temperature of the soup over the time interval $0 \leq t \leq 12$ minutes?

- A) $116^{\circ}F$
- B) $139^{\circ}F$
- C) $186^{\circ}F$
- D) $194^{\circ}F$

45. The value of the improper integral

$$\int_1^{\infty} e^{-x^2} dx$$

is best approximated by

- A) 0.006
- B) 0.14
- C) 0.28
- D) 0.74

MATHCH

END OF PART B OF SECTION I

Section II: Free-Response Questions

Directions: Answer each of the following questions in the space provided. Show all work clearly and in an organized manner. Correct reasoning and appropriate use of mathematical notation are required for full credit.

Justify your answers whenever requested. This may include showing calculations, explaining reasoning, or stating relevant definitions or theorems.

If an answer cannot be determined exactly, an appropriate numerical approximation may be used unless otherwise stated. Calculators may be used only if the question allows it.

Manage your time carefully and try not to spend too much time on any single question.

Important Notes:

Answers without supporting work or justification may receive little or no credit.

Clearly labeled diagrams or graphs may be used to support your reasoning.

Write neatly and clearly so that your work can be easily followed.

Section II, Part A

Time - 30 minutes, Number of questions - 2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS

1. People enter a line for a ferris wheel ride at a rate modeled by the function $R(t)$. The rate $R(t)$, in people per minute, is given by

$$R(t) = \begin{cases} 50 \left(\frac{t}{120} \right)^3 \left(1 - \frac{t}{400} \right)^6, & 0 \leq t \leq 400 \\ 0, & 400 \leq t \end{cases}$$

Time t is measured in minutes after the park opens.

People leave the line at a constant rate of 0.6 person per minute, as they board the ferris wheel.

At opening time, $t = 0$, there are 30 people already waiting in line.

- a) How many people enter the line for the ferris wheel during the first 400 minutes after the park opens? Give your answer to the nearest whole number, and include units.
- b) During the time interval $0 \leq t \leq 400$, there are always people waiting in line. How many people are in line at time $t = 400$? Give your answer to the nearest whole number, and include units.
- c) For $t > 400$, no new people enter the line (because $R(t) = 0$ for $t > 400$). As people continue to leave at a constant rate of 0.6 person per minute, at what time t is the line first empty? Give your answer to the nearest whole number.
- d) For $0 \leq t \leq 400$, at what time t is the number of people in line a minimum? Give your answer to the nearest whole number. Justify your answer by analyzing the sign of the derivative of the number of people in line, $N'(t)$, or by using a valid second-derivative test. Also find the minimum number of people in line, to the nearest whole number.

2. A particle moves in the plane so that its position at time t , for $0 \leq t \leq 4$, is given by

$$x(t) = t + \ln(t + 1), \quad y(t) = t + \sin t.$$

a) Find $\frac{dy}{dx}$ at $t = 1$.

b) Find the equation of the line tangent to the path of the particle at $t = 1$.

c) Find the value of t in the interval $0 < t \leq 4$ for which the line tangent to the curve is horizontal.

Show the equation you solve, and give the value of t to the nearest thousandth.

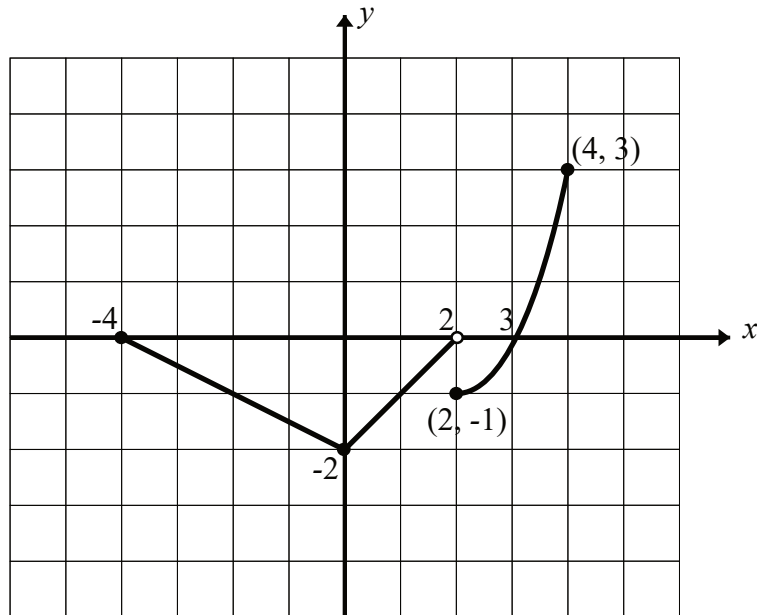
d) Find the total distance traveled by the particle from $t = 0$ to $t = 4$. Use a calculator to evaluate the distance to the nearest thousandth.

END OF PART A OF SECTION II

Section II, Part B

Time - 1 hour, Number of questions - 4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS



3. The graph of the continuous function g , the derivative of the function f , is shown above. The function g is piecewise linear for $-4 \leq x < 2$, and $g(x) = (x - 2)^2 - 1$ for $2 \leq x \leq 5$.
- If $f(0) = 4$, what is the value of $f(-4)$?
 - Evaluate $\int_0^5 g(x) dx$.
 - For $-4 < x < 5$, on what open intervals, if any, is the graph of f both decreasing and concave down? Give a reason for your answer.
 - Find the x -coordinate of each point of inflection of the graph of f . Give a reason for your answer.

4. The height of a plant at time t years is given by a twice-differentiable function P , where $P(t)$ is measured in centimeters and t in years. Selected values of $P(t)$ are shown in the table below:

t (years)	1	2	4	6	9
$P(t)$ (cm)	12	20	45	78	120

- a) Use the data in the table to estimate $P'(5)$. Using correct units, interpret the meaning of $P'(5)$ in the context of the problem.
- b) Explain why there must be at least one time t , for $1 < t < 9$, such that $P'(t) = 10$.
- c) Use a trapezoidal sum with the four subintervals indicated by the data to approximate the average height of the plant over the time interval $1 \leq t \leq 9$.
- d) The height of the plant (in centimeters) can also be modeled by

$$Q(x) = \frac{200x}{x+5},$$

where x is the width of the stem (in centimeters). When the plant is 100 cm tall, the stem's width increases at a rate of 0.04 cm per year. According to this model, what is the rate of change of the height of the plant with respect to time (in cm/year) when the plant is 100 cm tall?

5. A function f is defined by the power series

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}.$$

a) Show that $f'(x) = \frac{1}{x+1}$ for $|x| < 1$.

b) Find the interval of convergence of the series for $f(x)$. Justify your answer.

c) Write the first four nonzero terms of the Maclaurin series for $f(x)$. Then use those terms to approximate $f(0.2)$.

d) Using an alternating series error bound, find an upper bound for the error when $f(0.2)$ is approximated using the four nonzero terms from part (c).

6. A medication is administered to a patient. The amount of medication in the bloodstream at time t minutes is modeled by a function M . At time $t = 0$, there are 94 milligrams of the medication in the bloodstream. For all $t > 0$, the amount remains greater than 30 milligrams.

The function M satisfies the differential equation

$$\frac{dM}{dt} = -\frac{1}{4}(M - 30),$$

where $M(t)$ is measured in milligrams.

a) Write an equation for the line tangent to the graph of M at $t = 0$. Use this equation to approximate $M(3)$.

b) Use $\frac{d^2M}{dt^2}$ to determine whether your approximation in part (a) is an underestimate or an overestimate of the actual amount of medication at $t = 3$. Give a reason for your answer.

c) For $0 \leq t < 4$, an alternate model for the amount of medication is given by a function N that satisfies

$$\frac{dN}{dt} = -3(N - 30)^{2/3}, \quad N(0) = 94.$$

Find an expression for $N(t)$. Based on this model, find $N(3)$.

END OF PART B OF SECTION II

ANSWERS

Note: To help keep the cost of this book as low as possible, complete worked solutions and AP-style scoring guidelines are delivered digitally via QR codes rather than printed on these pages.

Practice Exam -1-

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	C	A	B	B	C	A	C	C	A	D	A	B	C	C
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
B	C	D	C	B	B	C	A	A	D	B	D	B	A	D
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
C	D	A	B	B	B	C	B	D	A	C	C	D	C	B

FRQ

Scan for solutions



Question 1

- (a) 882 people
- (b) 672 people
- (c) $t = 1520$ minutes
- (d) Minimum at $t = 33$ minutes and minimum number = 16 people

Scoring Breakdown:

2 - 2 - 2 - 3 (Total 9 points)

Question 2

- (a) ≈ 1.027
- (b) $y - (1 + \sin 1) = \frac{2(1 + \cos 1)}{3}(x - (1 + \ln 2))$
- (c) Solve $1 + \cos t = 0$. $t = \pi \approx 3.142$
- (d) ≈ 6.828

Scoring Breakdown:

2 - 2 - 2 - 3 (Total 9 points)

Question 3

- (a) 8
- (b) 4
- (c) $(-4, 0)$
- (d) $x = 0$

Scoring Breakdown:

2 - 2 - 2 - 3 (Total 9 points)

Question 4

- (a) 16.5 cm/year
- (b) By the Mean Value Theorem, there exists t such that $P'(t) = 10$
- (c) $501/8$ cm or 62.625 cm
- (d) 0.4 cm/year

Scoring Breakdown:

2 - 2 - 3 - 2 (Total 9 points)

Question 5

- (a) $f'(x) = \frac{1}{x+1} (|x| < 1)$
- (b) $(-1, 1]$
- (c) $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$ $f(0.2) \approx 0.1822667$
- (d) 0.000064

Scoring Breakdown:

2 - 2 - 3 - 2 (Total 9 points)

Question 6

- (a) 46 mg
- (b) The approximation is an underestimate. ($M'(t) < 0$, $M''(t) > 0$)
- (c) 31mg

Scoring Breakdown:

3 - 2 - 4 (Total 9 points)