

AP[®] Calculus

BC Workbook

- Module 1 - Parametric & Polar Analysis
- Module 2 - Integration Techniques
- Module 3 - Differential Equations
- Module 4 - Convergence of Infinite Series
- Module 5 - Power Series & Taylor Analysis

750+
Questions

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AP CALCULUS BC — WORKBOOK

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MODULE 4

CONVERGENCE OF INFINITE SERIES

SECTION I

Nth Term Test & Geometric Series

SECTION II

p-Series & Integral Test

SECTION III

Direct & Limit Comparison Tests

SECTION IV

Ratio Test

SECTION V

Root Test

SECTION VI

Alternating Series Test (AST)

SECTION VII

Absolute vs Conditional Convergence

SECTION 1 – Nth Term Test & Geometric Series

1. Nth Term Test (Divergence Test)

For a series

$$\sum a_n$$

evaluate

$$\lim_{n \rightarrow \infty} a_n$$

Rules

If

$$\lim a_n \neq 0$$

→ the series diverges

If

$$\lim a_n = 0$$

→ test is inconclusive

Important:

$$\lim a_n = 0$$

does not guarantee convergence.

Example:

$$\sum \frac{1}{n}$$

diverges.

2. Geometric Series

A geometric series has the form

$$\sum_{n=0}^{\infty} ar^n$$

Convergence Rule

$$|r| < 1$$

→ converges

$$|r| \geq 1$$

→ diverges

Sum Formula

If

$$|r| < 1$$
$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

Example

$$\sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n = \frac{1}{1-\frac{1}{3}} = \frac{3}{2}$$

Drill Set A – Nth Term Test

Determine whether the series diverges using the Nth Term Test.

1.

$$\sum \frac{n}{n+1}$$

2.

$$\sum \frac{2n+3}{n}$$

3.

$$\sum \frac{\sin n}{n}$$

4.

$$\sum \frac{n^2}{n^2+1}$$

5.

$$\sum \frac{n}{n^2 + 1}$$

6.

$$\sum \frac{\ln n}{n}$$

Drill Set B – Geometric Series

Determine convergence. If the series converges, find the sum.

1.

$$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$$

2.

$$\sum_{n=1}^{\infty} 3 \left(\frac{1}{4}\right)^n$$

3.

$$\sum_{n=0}^{\infty} (-0.6)^n$$

4.

$$\sum_{n=1}^{\infty} 5 \left(\frac{2}{3}\right)^n$$

5.

$$\sum_{n=0}^{\infty} 2(-0.5)^n$$

6.

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$$

SECTION MCQ

1

If

$$\lim_{n \rightarrow \infty} a_n = 2$$

then the series

$$\sum a_n$$

- (A) converges
 - (B) diverges
 - (C) conditional
 - (D) inconclusive
-

2

Consider

$$\sum \frac{n}{n+5}$$

Using the divergence test the series

- (A) converges
 - (B) diverges
 - (C) conditional
 - (D) geometric
-

3

Which series is geometric?

- (A) $\sum \frac{1}{n^2}$
- (B) $\sum \left(\frac{3}{4}\right)^n$
- (C) $\sum \frac{n}{n+1}$
- (D) $\sum \ln n$

4

A geometric series converges when

- (A) $r > 1$
 - (B) $|r| < 1$
 - (C) $r = 1$
 - (D) $r = 0$
-

5

The sum of

$$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$$

is

- (A) 1
 - (B) 2
 - (C) 3
 - (D) diverges
-

6

The series

$$\sum_{n=0}^{\infty} \left(\frac{5}{4}\right)^n$$

- (A) converges
 - (B) diverges
 - (C) conditional
 - (D) geometric
-

7

Which test should always be applied first?

- (A) Ratio Test
 - (B) Root Test
 - (C) Divergence Test
 - (D) Integral Test
-

8

The series

$$\sum_{n=0}^{\infty} (-0.5)^n$$

- (A) diverges
 - (B) converges absolutely
 - (C) conditional
 - (D) inconclusive
-

9 The series

$$\sum \frac{n}{n^2}$$

has $\lim a_n =$

- (A) 0
 - (B) 1
 - (C) ∞
 - (D) undefined
-

10

A geometric series diverges when

- (A) $|r| < 1$
- (B) $|r| > 1$
- (C) $r = 0$
- (D) $|r| = 0$

SECTION 2 – p-Series & Integral Test

1. The p-Series

A p-series has the form

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

Convergence Rule

$$\sum \frac{1}{n^p}$$

- Converges if $p > 1$
- Diverges if $p \leq 1$

2. The Integral Test

Let $f(x)$ be:

- Continuous
- Positive
- Decreasing
- Defined for $x \geq 1$

If $a_n = f(n)$, then

$$\sum a_n$$

and

$$\int_1^{\infty} f(x) dx$$

either both converge or both diverge.

Drill Set A – p-Series

Determine convergence:

1. $\sum \frac{1}{n^2}$

2. $\sum \frac{1}{n^{0.8}}$

3. $\sum \frac{1}{n^{1.01}}$

4. $\sum \frac{1}{n}$

5. $\sum \frac{1}{n^3}$

6. $\sum \frac{1}{\sqrt{n}}$

7. $\sum \frac{1}{n^{1.5}}$

8. $\sum \frac{1}{n^{0.99}}$

Drill Set B – Integral Test Application

Determine convergence using the Integral Test:

1. $\sum \frac{1}{n \ln n}$

2. $\sum \frac{\ln n}{n^2}$

3. $\sum \frac{1}{n(\ln n)^2}$

4. $\sum \frac{1}{n^{1.2}}$

5. $\sum \frac{\ln n}{n^{1.5}}$

6. $\sum \frac{1}{n^2+1}$

Section MCQ

1.

$$\sum_{n=1}^{\infty} \frac{1}{n^{1.2}}$$

- (A) Converges
 - (B) Diverges
 - (C) Alternating
 - (D) Inconclusive
-

2.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

- (A) Converges
 - (B) Diverges
 - (C) Alternating
 - (D) Inconclusive
-

3.

The p-series

$$\sum \frac{1}{n^p}$$

diverges when:

- (A) $p = 1$
- (B) $p > 1$
- (C) $p < 1$
- (D) both A and C

4.

The Integral Test requires:

- (A) alternating terms
 - (B) exponential growth
 - (C) factorial
 - (D) decreasing positive function
-

5.

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

Using Integral Test, the corresponding integral behaves like:

- (A) $\int 1/x^2 dx$
 - (B) $\int \ln x dx$
 - (C) $\int 1/(x \ln x) dx$
 - (D) $\int 1/x dx$
-

6.

$$\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$$

Compare with:

- (A) $1/n^2$
 - (B) $1/n$
 - (C) $1/n^{0.5}$
 - (D) $1/\ln n$
-

7.

If the improper integral converges and conditions hold, the series:

- (A) diverges
- (B) converges
- (C) oscillates
- (D) inconclusive

8.

$$\sum \frac{1}{n^{0.99}}$$

- (A) Converges
 - (B) Diverges
 - (C) Alternating
 - (D) Conditional
-

9.

$$\sum \frac{1}{n^2 + 1}$$

Best classification method:

- (A) p-series
 - (B) Integral Test
 - (C) Ratio
 - (D) Root
-

10.

$$\sum \frac{1}{n}$$

Integral comparison gives:

- (A) finite area
- (B) bounded
- (C) alternating
- (D) infinite area

SECTION 3 – Direct & Limit Comparison Test

1. Direct Comparison Test

If

$$0 \leq a_n \leq b_n$$

and

$$\sum b_n$$

converges, then

$$\sum a_n$$

also converges.

If

$$a_n \geq b_n \geq 0$$

and

$$\sum b_n$$

diverges, then

$$\sum a_n$$

diverges.

2. Limit Comparison Test

If

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$$

where $0 < L < \infty$,

then both series behave the same.

Drill Set A – Direct Comparison

1. $\sum \frac{1}{n^2+1}$

2. $\sum \frac{3n}{n^3+1}$

3. $\sum \frac{1}{n+\sqrt{n}}$

4. $\sum \frac{2}{n^2+n}$

5. $\sum \frac{5}{n^3-1}$

$$6. \sum \frac{1}{n^{0.8} + n^2}$$

Drill Set B – Limit Comparison

$$1. \sum \frac{n}{n^3 + 1}$$

$$2. \sum \frac{2n^2 + 1}{3n^2 + 5}$$

$$3. \sum \frac{n^2}{n^3 + 4}$$

$$4. \sum \frac{\ln n}{n}$$

5. $\sum \frac{n+1}{n^2+3}$

6. $\sum \frac{5n}{n^2-1}$

Section MCQ

1.

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$$

Best comparison:

- (A) $1/n$
- (B) $1/n^2$
- (C) $1/n^3$
- (D) e^{-n}

2.

$$\sum_{n=1}^{\infty} \frac{3n}{n^3 + 1}$$

Dominant behavior matches:

- (A) $1/n$
- (B) $1/n^2$
- (C) $1/n^3$
- (D) constant

3.

$$\sum_{n=1}^{\infty} \frac{n^2 + 1}{5n^2 + 3}$$

This series:

- (A) converges
 - (B) conditional
 - (C) alternating
 - (D) diverges
-

4.

$$\sum_{n=1}^{\infty} \frac{1}{n + \sqrt{n}}$$

Best comparison:

- (A) $1/n$
 - (B) $1/n^2$
 - (C) $1/\sqrt{n}$
 - (D) e^{-n}
-

5.

$$\sum_{n=1}^{\infty} \frac{n}{n^2 + 4}$$

Limit Comparison should use:

- (A) n
- (B) $1/n$
- (C) $1/n^2$
- (D) $1/\sqrt{n}$

6.

If

$$0 \leq a_n \leq \frac{1}{n^2}$$

then

$$\sum a_n$$

- (A) converges
 - (B) diverges
 - (C) inconclusive
 - (D) oscillates
-

7.

$$\sum_{n=1}^{\infty} \frac{2}{n^{0.8} + n^2}$$

Dominant term is:

- (A) $n^{0.8}$
 - (B) exponential
 - (C) constant
 - (D) n^2
-

8.

$$\sum_{n=1}^{\infty} \frac{n^3 + 5}{n^4 + 2}$$

Behavior matches:

- (A) $1/n$
- (B) $1/n^2$
- (C) constant
- (D) $1/n^3$

9.

If

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 5$$

and $\sum b_n$ converges, then

- (A) $\sum a_n$ diverges
 - (B) inconclusive
 - (C) $\sum a_n$ converges
 - (D) oscillates
-

SECTION 4 – Ratio Test

Ratio Test

Let

$$L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

- If $L < 1$, converges absolutely
 - If $L > 1$, diverges
 - If $L = 1$, inconclusive
-

Drill

1. $\sum \frac{3^n}{n!}$

2. $\sum \frac{n!}{3^n}$

3. $\sum \frac{n^2}{2^n}$

4. $\sum \frac{2^n}{n^2}$

5. $\sum \frac{5^n}{n!}$

6. $\sum \frac{n!}{n^n}$

7. $\sum \frac{n^3}{4^n}$

8. $\sum \frac{7^n}{n^3}$

Section MCQ

1.

$$\sum_{n=1}^{\infty} \frac{3^n}{n!}$$

- (A) Converges absolutely
 - (B) Diverges
 - (C) Conditional
 - (D) Inconclusive
-

2.

$$\sum_{n=1}^{\infty} \frac{n!}{3^n}$$

- (A) Converges
 - (B) Diverges
 - (C) Alternating
 - (D) Inconclusive
-

3.

$$\sum_{n=1}^{\infty} \frac{n^2}{2^n}$$

- (A) Converges absolutely
- (B) Diverges
- (C) Conditional
- (D) Inconclusive

4.

$$\sum_{n=1}^{\infty} \frac{2^n}{n^2}$$

- (A) Converges
 - (B) Inconclusive
 - (C) Alternating
 - (D) Diverges
-

5.

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}$$

- (A) Converges
 - (B) Inconclusive
 - (C) Alternating
 - (D) Diverges
-

6.

If Ratio Test gives $L = 1$, then:

- (A) Converges
 - (B) Diverges
 - (C) Inconclusive
 - (D) Alternating
-

7.

$$\sum_{n=1}^{\infty} \frac{5^n}{n!}$$

- (A) Converges absolutely
- (B) Diverges
- (C) Conditional
- (D) Inconclusive

8.

$$\sum_{n=1}^{\infty} \frac{n^3}{4^n}$$

- (A) Converges
 - (B) Diverges
 - (C) Alternating
 - (D) Inconclusive
-

9.

$$\sum_{n=1}^{\infty} \frac{7^n}{n^3}$$

- (A) Converges
 - (B) Diverges
 - (C) Alternating
 - (D) Inconclusive
-

10.

Ratio Test is most effective when the series contains:

- (A) logarithm only
- (B) polynomial only
- (C) alternating sign
- (D) factorial or exponential

SECTION 5 – Root Test

Root Test

$$L = \lim_{n \rightarrow \infty} \sqrt[n]{|a_n|}$$

- $L < 1 \rightarrow$ converges
 - $L > 1 \rightarrow$ diverges
 - $L = 1 \rightarrow$ inconclusive
-

Drill

1. $\sum \frac{2^n}{n^n}$

2. $\sum \left(\frac{3n+1}{4n+2}\right)^n$

3. $\sum \frac{n^n}{5^n}$

4. $\sum \left(\frac{1}{n}\right)^n$

5. $\sum \frac{4^n}{n^n}$

6. $\sum \left(\frac{2n}{3n+1}\right)^n$

Section MCQ

1.

$$\sum_{n=1}^{\infty} \frac{2^n}{n^n}$$

- (A) Converges absolutely
 - (B) Diverges
 - (C) Conditional
 - (D) Inconclusive
-

2.

$$\sum_{n=1}^{\infty} \left(\frac{3n+1}{4n+2} \right)^n$$

- (A) Converges
 - (B) Diverges
 - (C) Alternating
 - (D) Inconclusive
-

3.

$$\sum_{n=1}^{\infty} \frac{n^n}{5^n}$$

- (A) Converges
 - (B) Diverges
 - (C) Alternating
 - (D) Inconclusive
-

4.

$$\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)^n$$

- (A) Converges
 - (B) Diverges
 - (C) Conditional
 - (D) Inconclusive
-

5.

$$\sum_{n=1}^{\infty} \frac{4^n}{n^n}$$

- (A) Converges
 - (B) Diverges
 - (C) Alternating
 - (D) Inconclusive
-

6.

$$\sum_{n=1}^{\infty} \left(\frac{2n}{3n+1}\right)^n$$

Limit of nth root equals:

- (A) 0
- (B) 3/2
- (C) 1
- (D) 2/3

7.

Root Test gives $L = 1$. Then:

- (A) Converges
 - (B) Diverges
 - (C) Inconclusive
 - (D) Alternating
-

8.

$$\sum_{n=1}^{\infty} \left(\frac{5n+1}{2n+4} \right)^n$$

- (A) Converges
 - (B) Diverges
 - (C) Conditional
 - (D) Inconclusive
-

9.

Root Test is most useful when:

- (A) factorial present
 - (B) polynomial only
 - (C) alternating only
 - (D) entire expression raised to power n
-

10.

$$\sum_{n=1}^{\infty} \left(\frac{n}{n+1} \right)^n$$

The n th root limit equals:

- (A) less than 1
- (B) 0
- (C) e
- (D) 1

$$3. \sum (-1)^n \frac{n}{n+1}$$

$$4. \sum (-1)^n \frac{1}{\sqrt{n}}$$

$$5. \sum (-1)^n \frac{1}{\ln n}$$

$$6. \sum (-1)^n \frac{1}{n^{1.5}}$$

Section MCQ

1.

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$$

- (A) Converges absolutely
 - (B) Converges conditionally
 - (C) Diverges
 - (D) Inconclusive
-

2.

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2}$$

- (A) Converges absolutely
 - (B) Converges conditionally
 - (C) Diverges
 - (D) Inconclusive
-

3.

If

$$\lim_{n \rightarrow \infty} b_n \neq 0$$

then

- (A) converges
- (B) diverges
- (C) conditional
- (D) absolute

4.

For AST to apply, the sequence b_n must:

- (A) be decreasing
 - (B) be increasing
 - (C) be alternating
 - (D) be factorial
-

5.

$$\sum (-1)^n \frac{n}{n+1}$$

- (A) Converges
 - (B) Diverges
 - (C) Conditional
 - (D) Absolute
-

6.

$$\sum (-1)^n \frac{1}{\sqrt{n}}$$

- (A) Converges absolutely
 - (B) Converges conditionally
 - (C) Diverges
 - (D) Inconclusive
-

7.

$$\sum (-1)^n \frac{1}{\ln n}$$

- (A) Converges
- (B) Diverges
- (C) Absolute
- (D) Inconclusive

8.

$$\sum (-1)^n \frac{1}{n^{1.5}}$$

- (A) Absolute
 - (B) Conditional
 - (C) Diverges
 - (D) Inconclusive
-

9.

If an alternating series converges but

$$\sum |a_n|$$

diverges, then it is:

- (A) Absolute
 - (B) Conditional
 - (C) Divergent
 - (D) Geometric
-

10.

The error after n terms of a convergent alternating series is bounded by:

- (A) a_n
- (B) limit value
- (C) $1/n$
- (D) a_{n+1}

SECTION 7 – Absolute vs Conditional Convergence

A series converges absolutely if

$$\sum |a_n|$$

converges.

If it converges but not absolutely \rightarrow conditional convergence.

Drill

1. $\sum (-1)^n \frac{1}{n}$

2. $\sum (-1)^n \frac{1}{n^2}$

3. $\sum (-1)^n \frac{1}{n^{0.9}}$

4. $\sum (-1)^n \frac{n}{n^2+1}$

5. $\sum (-1)^n \frac{\ln n}{n^2}$

6. $\sum (-1)^n \frac{1}{n \ln n}$

Section MCQ

1.

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$$

- (A) Absolute
 - (B) Conditional
 - (C) Divergent
 - (D) Inconclusive
-

2.

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2}$$

- (A) Absolute
 - (B) Conditional
 - (C) Divergent
 - (D) Inconclusive
-

3.

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^{0.8}}$$

- (A) Absolute
- (B) Conditional
- (C) Divergent
- (D) Inconclusive

4.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2 + 1}$$

Behavior matches:

- (A) constant
 - (B) exponential
 - (C) $1/n^2$
 - (D) $1/n$
-

5.

$$\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n^2}$$

- (A) Absolute
 - (B) Conditional
 - (C) Divergent
 - (D) Inconclusive
-

6.

$$\sum_{n=2}^{\infty} (-1)^n \frac{1}{n \ln n}$$

Absolute value series behaves like:

- (A) $1/n^2$
 - (B) $1/n$
 - (C) $1/(n \ln n)$
 - (D) constant
-

7.

If a series converges absolutely, it:

- (A) may diverge
- (B) is geometric
- (C) must be alternating
- (D) must converge

8.

If

$$\sum |a_n|$$

diverges but

$$\sum a_n$$

converges, the series is:

- (A) Absolute
 - (B) Conditional
 - (C) Divergent
 - (D) Alternating
-

9.

$$\sum (-1)^n \frac{1}{\sqrt{n}}$$

Absolute value behaves like:

- (A) geometric
 - (B) factorial
 - (C) p-series with $p=2$
 - (D) p-series with $p=1/2$
-

10.

Proper order of testing for alternating series:

- (A) AST first, absolute later
- (B) Ratio only
- (C) Root only
- (D) Absolute first, then AST

MODULE 4 – FINAL ASSESSMENT

Part I – Multiple Choice (20 Questions)

1.

$$\sum_{n=1}^{\infty} \frac{1}{n^{1.3}}$$

- (A) Converges
 - (B) Diverges
 - (C) Conditional
 - (D) Inconclusive
-

2.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

- (A) Converges
 - (B) Diverges
 - (C) Conditional
 - (D) Inconclusive
-

3.

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

- (A) Converges
- (B) Diverges
- (C) Absolute
- (D) Geometric

4.

$$\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$$

- (A) Converges
 - (B) Diverges
 - (C) Conditional
 - (D) Inconclusive
-

5.

$$\sum_{n=1}^{\infty} \frac{n}{n^3 + 1}$$

Best comparison:

- (A) constant
 - (B) $1/n$
 - (C) $1/n^2$
 - (D) $1/n^3$
-

6.

$$\sum_{n=1}^{\infty} \frac{n^2 + 1}{3n^2 + 5}$$

- (A) Converges
 - (B) Diverges
 - (C) Conditional
 - (D) Inconclusive
-

7.

$$\sum_{n=1}^{\infty} \frac{3^n}{n!}$$

- (A) Absolute
 - (B) Divergent
 - (C) Conditional
 - (D) Inconclusive
-

8.

$$\sum_{n=1}^{\infty} \frac{n!}{4^n}$$

- (A) Converges
 - (B) Diverges
 - (C) Conditional
 - (D) Inconclusive
-

9.

$$\sum_{n=1}^{\infty} \frac{n^3}{5^n}$$

- (A) Converges
 - (B) Diverges
 - (C) Alternating
 - (D) Inconclusive
-

10.

If Ratio Test gives $L = 1$, then:

- (A) Converges
 - (B) Diverges
 - (C) Inconclusive
 - (D) Absolute
-

11.

$$\sum_{n=1}^{\infty} \left(\frac{2n}{3n+1} \right)^n$$

- (A) Converges
 - (B) Diverges
 - (C) Conditional
 - (D) Inconclusive
-

12.

$$\sum_{n=1}^{\infty} \left(\frac{5n+1}{2n+4} \right)^n$$

- (A) Converges
 - (B) Diverges
 - (C) Absolute
 - (D) Inconclusive
-

13.

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$$

- (A) Absolute
 - (B) Conditional
 - (C) Divergent
 - (D) Inconclusive
-

14.

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^{1.5}}$$

- (A) Absolute
 - (B) Conditional
 - (C) Divergent
 - (D) Inconclusive
-

15.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+1}$$

- (A) Converges
 - (B) Diverges
 - (C) Absolute
 - (D) Inconclusive
-

16.

$$\sum_{n=1}^{\infty} \frac{4^n}{n^n}$$

- (A) Converges
- (B) Diverges
- (C) Conditional
- (D) Inconclusive

17.

$$\sum_{n=1}^{\infty} \frac{2^n}{n^n}$$

- (A) Converges
 - (B) Diverges
 - (C) Conditional
 - (D) Inconclusive
-

18.

If $\sum |a_n|$ converges, then:

- (A) conditional
 - (B) alternating
 - (C) $\sum a_n$ diverges
 - (D) $\sum a_n$ converges
-

19.

$$\sum_{n=1}^{\infty} \frac{\ln n}{n}$$

- (A) Converges
 - (B) Diverges
 - (C) Absolute
 - (D) Inconclusive
-

20.

Correct testing order for alternating series:

- (A) Ratio only
 - (B) Root only
 - (C) AST \rightarrow Absolute
 - (D) Absolute \rightarrow AST
-

SERIES CONVERGENCE

DECISION FLOW SYSTEM

STEP 0 — Divergence Test (ALWAYS FIRST)

$$\lim_{n \rightarrow \infty} a_n$$

- If limit $\neq 0 \rightarrow$ **DIVERGES**
 - If limit = 0 \rightarrow continue
-

STEP 1 — Recognize the Pattern

Geometric Form?

$$\sum ar^n$$

Converges if:

$$|r| < 1$$

p-Series?

$$\sum \frac{1}{n^p}$$

Converges if:

$$p > 1$$

Alternating?

$$\sum (-1)^n b_n$$

If yes:

1. Test absolute convergence
 2. If absolute fails \rightarrow apply AST
-

▣ STEP 2 — Structural Identification

□ Factorial or Exponential Present?

Examples:

$$\frac{3^n}{n!}, \frac{n!}{4^n}$$

→ Use **Ratio Test**

$$L = \lim \left| \frac{a_{n+1}}{a_n} \right|$$

- $L < 1$ → Converges
 - $L > 1$ → Diverges
 - $L = 1$ → Inconclusive
-

□ Entire Expression Raised to Power n?

Example:

$$\left(\frac{2n}{3n+1} \right)^n$$

→ Use **Root Test**

$$L = \lim \sqrt[n]{|a_n|}$$

Same interpretation as Ratio Test.

□ Rational Function in n?

Example:

$$\frac{n^2 + 1}{3n^3 + 4}$$

- Compare dominant powers
- Use Direct or Limit Comparison

□ Logarithm Involved?

Example:

$$\frac{\ln n}{n}$$

→ Try Integral Test

→ Or Limit Comparison with $1/n$

▣ STEP 3 — Absolute vs Conditional

If alternating:

1. Evaluate

$$\sum |a_n|$$

2. If converges → **Absolute Convergence**

3. If diverges but AST applies → **Conditional Convergence**
-

▣ STEP 4 — AST Requirements

For

$$\sum (-1)^n b_n$$

Must verify:

- $b_n > 0$
 - b_n decreasing
 - $\lim b_n = 0$
-

▣ WARNING ZONES

⚠ Ratio/Root gives $L = 1$ → try another test

⚠ Do not compare with wrong dominant term

△ Integral Test requires decreasing

△ AST does NOT test absolute convergence

Quick Structural Guide

Structure	Primary Tool
$1/n^p$	p-test
factorial	Ratio
c^n	Ratio
$(\dots)^n$	Root
rational in n	Comparison
$\ln(n)/n$	Integral
alternating	Absolute → AST

FINAL EXAM STRATEGY

1. Divergence Test
2. Simplify dominant term
3. Choose structural test
4. If alternating → check absolute first
5. If stuck → try comparison

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